CORE MATHEMATICS (C) UNIT 2 TEST PAPER 2

1. Find the first three terms in the binomial expansion, in ascending powers of x, of

$$\left(1-\frac{x}{2}\right)^5.$$ [5]

2. Find $\int_1^9 (1+2x+\sqrt{x})\ dx$. [5]

3. Given that $\sin 3\theta = \cos 3\theta$,

(i) state the value of $\tan 3\theta$. [1]

(ii) Hence find the values of θ, in the interval $0 \leq \theta \leq 2\pi$, for which $\sin 3\theta = \cos 3\theta$. [5]

4. The first three terms of an arithmetic series are 21.5, 20, 18.5.

(i) Find the smallest value of n for which the nth term of the series is negative. [4]

(ii) For this value of n, find the sum of the first n terms. [3]

5. Given that $f(x) \equiv \cos x$, sketch on separate diagrams for $-2\pi \leq x \leq 2\pi$ the curves with the following equations. In each case show the coordinates of all points of intersection with the coordinate axes and all maximum and minimum points.

(i) $y = k\,f(x)$, where $k > 0$, [3]

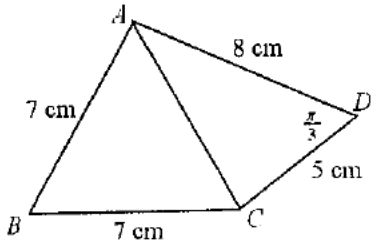
(ii) $y = f(x - a)$, where $0 < x < \frac{\pi}{4}$. [4]

6. A shrub is planted when it is 2 m tall. In the nth year after planting, the shrub grows in height by h_n m, where $h_{n+1} = 0.8\ h_n$. One year after planting, it is 2.3 m tall.

(i) Find the height of the shrub after 10 years, in m to 2 decimal places. [4]

(ii) Show that the shrub will never grow to more than 3.5 m in height. [3]

7. In the diagram, $AB = BC = 7$ cm, $CD = 5$ cm, $AD = 8$ cm and angle $ADC = \frac{\pi}{3}$ radians.



(i) Show that angle $ABC = \frac{\pi}{3}$ radians. [4]

With centres A and C, arcs BC and AB are drawn.

(ii) Find the area of the figure bounded by these two arcs and the straight lines AD and DC. [5]

8. (i) Given that $3 \log_2 x - 2 \log_2 y = 3$, show that $x^3 = 8y^2$. [3]

(ii) Given also that $2 \log_3 x = \log_3 y + 1$, express x^2 in terms of y. [2]

(iii) Deduce that if $3 \log_2 x - 2 \log_2 y = 3$ and $2 \log_3 x = \log_3 y + 1$, then $4y^{1/3} = 3$. [3]

(iv) Hence find the real value of x which satisfies these two equations. [3]

9. (i) Use the Factor Theorem to show that $(x + 2)$ is a factor of $x^3 - 4x^2 - 3x + 18$. [2]

(ii) Factorise $x^3 - 4x^2 - 3x + 18$ completely. [3]

(iii) Show that the x-axis is a tangent to the curve C with equation $y = x^3 - 4x^2 - 3x + 18$, and state the point at which tangency occurs. [2]

(iv) Sketch the curve C. [3]

(v) Calculate the area of the finite region bounded by C and the x-axis. [5]

CORE MATHS 2 (C) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1. $(1 - x/2)^5 = 1 - 5x/2 + (5)(4)(-x/2)^2/2 = 1 - 5x/2 + 5x^2/2$ — M1 A1 M1 A1 A1 — 5

2. $\int_1^9 (1 + 2x + x^{1/2})\,dx = \left[x + x^2 + \frac{2}{3}x^{3/2}\right]_1^9 = [90 + 18] - [2 + \frac{2}{3}] = 105\frac{1}{3}$ — B1 M1 A1 M1 A1 — 5

3. (i) $\tan 3\theta = 1$ (ii) $3\theta = \pi/4, 5\pi/4, 9\pi/4, 13\pi/4, 17\pi/4, 21\pi/4$ — B1 M1 A1 A1
   $\theta = \pi/12, 5\pi/12, 3\pi/4, 13\pi/12, 17\pi/12, 7\pi/4$ — M1 A1 — 6

4. (i) $T_n = 21.5 + (n - 1)(-1.5) = 23 - 1.5n < 0$ when $n > 46/3$, so $n = 16$ — M1 A1 M1 A1
   (ii) $S_{16} = 8(43 + 15(-1.5)) = 8(43 - 22.5) = 8(20.5) = 164$ — M1 A1 A1 — 7

5. (i) Curve through $(\pm 2\pi, k)$, $(\pm 3\pi/2, 0)$, $(\pm\pi, -k)$, $(\pm\pi/2, 0)$, $(0, k)$ — B3
   (ii) $(-2\pi + a, 1)$, $(\pm 3\pi/2 + a, 0)$, $(\pm\pi + a, -1)$, $(\pm\pi/2 + a, 0)$, $(a, 1)$, $(0, \cos a)$ — B4 — 7

6. (i) Height $= 2 + 0.3 + 0.24 + \ldots 2 + 0.3(1 - 0.8^{10})/(1 - 0.8) = 3.34$ m — B1 M1 A1 A1
   (ii) Sum to infinity $= 2 + 0.3/(1 - 0.8) = 3.5$ m — M1 A1 A1 — 7

7. (i) $AC^2 = 25 + 64 - 80\cos\pi/3 = 49$ $AC = 7$ cm — M1 A1
   Triangle ABC is equilateral, so angle $ABC = \pi/3$ — M1 A1
   (ii) Sector $ABC = 49\pi/6$ Triangle ABC + 2 sectors = — B1 M1
   $49\pi/3 - 49\sqrt{3}/4$ Total area $= 49\pi/3 - 9\sqrt{3}/4 \approx 47.4$ cm^2 — A1 M1 A1 — 9

8. (i) $\log_2 (x^3/y^2) = 3$ $x^3/y^2 = 2^3$ $x^3 = 8y^2$ — M1 A1 A1
   (ii) $\log_3 (x^2/y) = 1$ $x^2/y = 3$ $x^2 = 3y$ — M1 A1
   (iii) $(8y^2)^{2/3} = 3y$ $4y^{4/3} = 3y$ $y \neq 0$, so $4y^{1/3} = 3$ — M1 A1 A1
   (iv) $y = (3/4)^3 = 27/64$ $x^2 = 81/64$ $x > 0$, so $x = 9/8$ — M1 A1 A1 — 11

9. (i) $f(-2) = -8 - 16 + 6 + 18 = 0$, so $(x + 2)$ is a factor — M1 A1
   (ii) $(x + 2)(x^2 - 6x + 9) = (x + 2)(x - 3)^2$ — B1 M1 A1
   (iii) Repeated factor $(x - 3)$, so curve touches x-axis at $(3, 0)$ — M1 A1
   (iv) Curve cutting axes at $(-2, 0)$, $(0, 18)$, $(3, 0)$ — B3
   (v) $\int_{-2}^{3} y\,dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} - \frac{3x^2}{2} + 18x\right]_{-2}^{3} = \frac{65}{4} - \frac{140}{3} - \frac{15}{2} + 90 = \frac{625}{12}$ — M1 A1 M1 A1 A1 — 15